NEAR MODELS FOR LONGITUDINAL DATA

$Y = X\beta + \varepsilon$, $Y = (Y_{ij})_{\substack{i=1,m \\ j=1,n}}$

$\varepsilon \sim MVN(0\ V)$, V = known

Log likelihood

$$L(\beta) = \tfrac{1}{2} n \log(2\pi)\ \tfrac{1}{2} \log|V|$$
$$- \tfrac{1}{2}(Y - X\beta)' V^{-1} (Y\ X\beta)$$

Where does this come from ?

→ if $Y \sim N(\mu\ \sigma^2)$ (random variable)

then $f_Y(y) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$

→ if $Y \sim MVN(\mu, V)$ $Y = (Y,\ ,\ Y_n)'$

then $f_Y(y) = \dfrac{}{\sqrt{(2\pi)^n\ V|}} e^{-\frac{1}{2}(Y\ X\beta)' V^{-1} (Y\ X\beta)}$

then $L(\beta) = \ g f_Y(y) =$

(2)

$f_Y(y)$ p.d.f = probability density fct

$f_Y(\beta)$ = likelihood fct

$L(\beta) = \log f_Y(\beta) = \log$ likelihood fct.

MLE $\hat{\beta}$ is obtained by maximizing $L(\beta)$

i.e minimizing

$$RSS = (Y - X\beta)' V^{-1} (Y - X\beta)$$

weighted sum of squared errors

$\hat{\beta}$ = WLS estimator

V known

general $\Rightarrow$ GLS

independent data $\Rightarrow$ WLS

arameters models
for the cov structure

$$Y_{ij} = X_{ij}\beta + \underbrace{U_i + W_i(t_{ij}) + Z_{ij}}_{\varepsilon_{ij}}$$

U_i RE

W_i serial process

Z_{ij} measurement error

$E(Y) = X'\beta$

$V(Y) = V(t_i)$

$V(\rho)$

$U \sim N(0, \nu^2)$ indep.

$W_i \sim N(0, \delta^2)$

$\mathrm{corr}(W_i(t)\ W_i(t-u)) = \rho(u) = \alpha^{|u|}$

$\mathrm{cov}(W(t), W_i(t-u)) = \delta^2(u)$

$\delta^2 = \delta^2(0) = \mathrm{Var}(W_i(t))$

$Z_i \sim N(0, z^2)$ $\quad V(\varepsilon_i) = \nu^2 + \delta^2 + z^2$

Interpretation

U_i R_1 - subject-specific intercept

ν^2 long-run between-subject var

$W_i(t_{ij})$ — $\mathrm{cov}(W_i(t), W_i(t-u)) = \delta^2(u)$

$\delta^2(u) \to 0$ as $u \to \infty$

~~U_i~~ δ^2 transient between-subject var

Z_{ij} = ME $\quad \mathrm{Var}(Z_i) = \tau^2$

ρ = within-subject corr

$\mathrm{corr}(W_i(t), W_i\, t\ u)) = \rho(u)$

VARIOGRAM

$$\gamma(u) = \tau^2 + \delta^2(1-\rho(u))$$

$$Var(\varepsilon_{ij}) = \nu^2 + \delta^2 + \tau^2$$

$$Var(Z_{ij}) = \tau^2$$

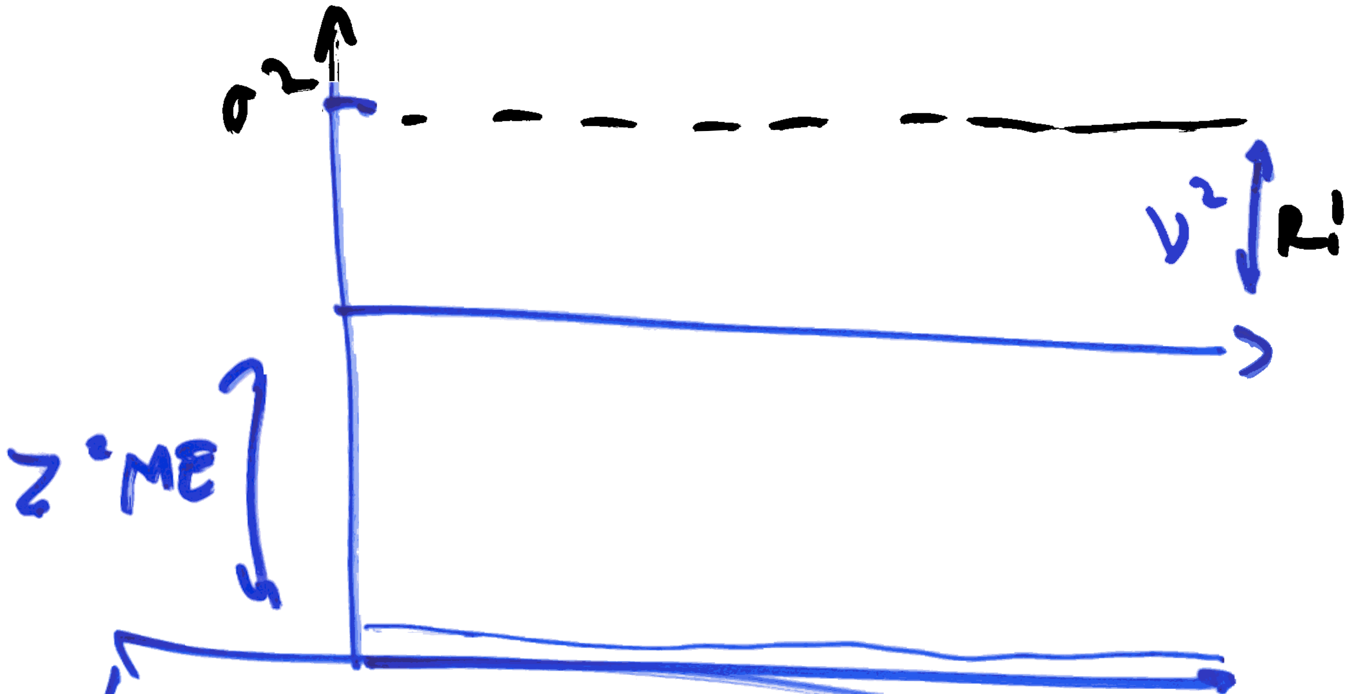
$$Var(U_i) = \nu^2$$

$$Var(W_i) = \delta^2$$

$$Corr(W_i(t), W_i(t-u)) = \rho(u)$$

$u \to 0: \rho(u) \to 1 \Rightarrow \gamma(u) \to \tau^2$

$u \to \infty \Rightarrow \gamma(u) \to \delta^2 + \tau^2$



$$\boxed{Y_{ij} = X_{ij}'\beta + U_i + \varepsilon_{ij}}$$

- uniform corr. structure + measurement error
- random intercept + ME